

Impact parameter dependence of π^-/π^+ ratio in probing the nuclear symmetry energy using heavy-ion collisions

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The impact parameter dependence of π^-/π^+ ratio is examined in heavy-ion collisions at 400MeV/nucleon within a transport model. It is shown that the sensitivity of π^-/π^+ ratio on symmetry energy shows a transition from central to peripheral collisions, i.e., the stiffer symmetry energy leads to a larger π^-/π^+ ratio in peripheral collisions while the softer symmetry energy always leads this ratio to be larger in central collisions. After checking the kinematic energy distribution of π^-/π^+ ratio, we found this transition of sensitivity of π^-/π^+ ratio to symmetry energy is mainly from less energetic pions, i.e., the softer symmetry energy gets the less energetic pions to form a smaller π^-/π^+ ratio in peripheral collisions while these pions generate a larger π^-/π^+ ratio in central collisions. Undoubtedly, the softer symmetry energy can also lead more energetic pions to form a larger π^-/π^+ ratio in peripheral collisions. Nevertheless, considering that most of pions are insufficient energetic at this beam energy, we therefore suggest the π^-/π^+ ratio as a probe of the high-density symmetry energy effective only in central at most to midcentral collisions, thereby avoiding the possible information of low-density symmetry energy carried in π^-/π^+ ratio from peripheral collisions.

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The determination of density-dependent nuclear symmetry energy is one of the hot topic in isospin physics due to its importance in understanding the structure of radiative nuclei in nuclear physics [1–4] and the evolution of massive stars and properties of neutron stars in nuclear astrophysics [5–9]. Presently, although many useful experimental observables [10–19] have been proposed to determine the nuclear symmetry energy, the knowledge regarding the nuclear symmetry energy is still far lack except for the relative determination of nuclear symmetry energy at saturation density ρ_0 from empirical liquid-drop mass formula [13, 20]. For example, by comparing the π^-/π^+ ratio with the FOPI experimental data [21], the Boltzmann-Uehling-Uhlenbeck (BUU) [22] and Boltzmann-Langevin (BL) [23] communities favor a super-soft symmetry energy, but the quantum molecular dynamics (QMD) [24] community suggests a super-stiff symmetry energy. Therefore, much more efforts need to be done to better determine the nuclear symmetry energy at both supersaturation and subsaturation densities.

Heavy-ion collisions induced by neutron-rich nuclei as an important tool are commonly used to study the density dependence of nuclear symmetry energy [25–30]. Usually, a higher compressive density formed in central heavy-ion collisions with the softer symmetry energy gets the π^-/π^+ ratio to be larger compared to the case of stiffer symmetry energy. However, the densities formed in heavy-ion collisions always experience a broad range

from subsaturation to supersaturation densities. Therefore, one has to evaluate the influence of the high-density (low-density) matter phase on observable when probing the symmetry energy at subsaturation (supersaturation) density using heavy-ion collisions due to the formation of supersaturation (subsaturation) density matter. Certainly, the influence of low-density matter phase is inevitable using heavy-ion collisions to probe high-density symmetry energy due to the densities formed at the final reaction stage is always lower than the saturation density, therefore, one has to select those of reaction production without experiencing the final reaction stage such as pre-equilibration neutron-proton ratio. On the other hand, as shown recently, the pion potential has an opposite effects on π^-/π^+ ratio compared to the effect of symmetry energy on it, and thus decreases the sensitivity of π^-/π^+ ratio to symmetry energy [31, 32]. Moreover, the modification of pion production threshold can even invert the sensitivity of π^-/π^+ ratio to symmetry energy [33]. Actually, impact parameter as a factor can also influence the compressive density of participating region and thus may even invert the sensitivity of π^-/π^+ ratio to symmetry energy in peripheral collisions as mentioned in our recent work about the influence of neutron-skin thickness on the π^-/π^+ ratio in heavy-ion collisions [34]. Therefore, it is necessary to systematically check the impact parameter dependence of π^-/π^+ ratio in probing the symmetry energy using heavy-ion collisions, and show the corresponding reasons and which energy range of pion does get the sensitivity of π^-/π^+ ratio to symmetry energy reversal. This is the main purpose of the present study.

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The present study is based on an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model [35]. In this model, an isospin-dependent mean-field is used to model the nuclear interaction, its expression is defined as follows:

$$\begin{aligned}
 U(\rho, \delta, \vec{p}, \tau) = & A_u(x) \frac{\rho_{-\tau}}{\rho_0} + A_l(x) \frac{\rho_{\tau}}{\rho_0} \\
 & + B \left(\frac{\rho}{\rho_0} \right)^{\sigma} (1 - x \delta^2) - 8\tau x \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^{\sigma}} \delta \rho_{-\tau} \\
 & + \frac{2C_{\tau, \tau}}{\rho_0} \int d^3 p' \frac{f_{\tau}(\vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} \\
 & + \frac{2C_{\tau, -\tau}}{\rho_0} \int d^3 p' \frac{f_{-\tau}(\vec{p}')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2}. \quad (1)
 \end{aligned}$$

In the above, $\rho = \rho_n + \rho_p$ is the nucleon number density and $\delta = (\rho_n - \rho_p)/\rho$ is the isospin asymmetry of the nuclear medium; $\rho_{n(p)}$ denotes the neutron (proton) density, the isospin τ is 1/2 for neutrons and -1/2 for protons, and $f(\vec{p})$ is the local phase space distribution function. The expressions and values of the parameters $A_u(x)$, $A_l(x)$, σ , B , $C_{\tau, \tau}$, $C_{\tau, -\tau}$, and Λ can be found in Refs. [36, 37], and they lead to the binding energy of -16 MeV, incompressibility 212 MeV for symmetric nuclear matter, and symmetry energy $E_{\text{sym}}(\rho_0) = 30.5$ MeV at saturation density $\rho_0 = 0.16 \text{ fm}^{-3}$, respectively. While parameter x is used to mimic the different forms of symmetry energy predicted by various many-body theories without changing any properties of symmetric nuclear matter and the value of symmetry energy at saturation density $E_{\text{sym}}(\rho_0)$. Shown in Fig. 1 is the density dependence of symmetry energy with a softer setting $x=1$ and stiffer one $x=0$.

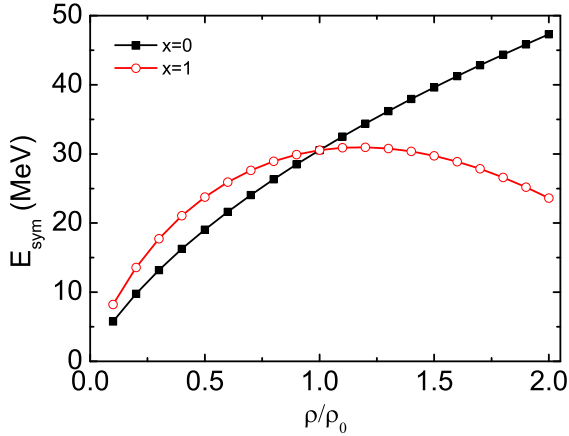


FIG. 1: The density dependence of nuclear symmetry energy.

Now let's check the impact parameter dependence of π^-/π^+ ratio in probing the symmetry energy. Within the IBUU transport model for heavy-ion collision at the intermediate energy, almost all the pions are produced from the decay of $\Delta(1232)$ resonances. Therefore, the

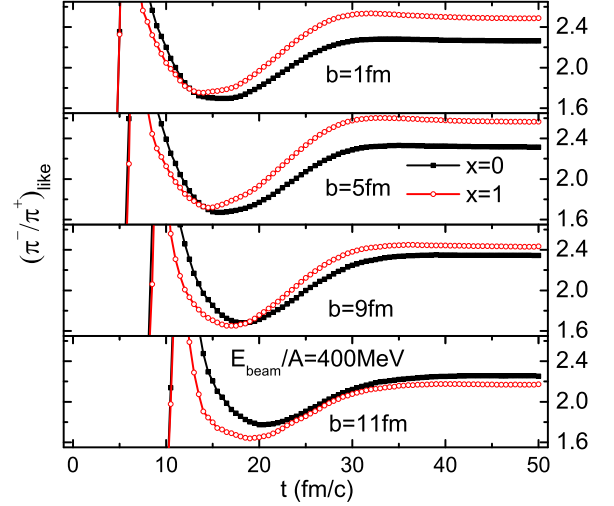


FIG. 2: The time evolution of $(\pi^-/\pi^+)_{\text{like}}$ ratio from central to peripheral Pb+Pb collisions at the beam energy of 400 MeV/nucleon.

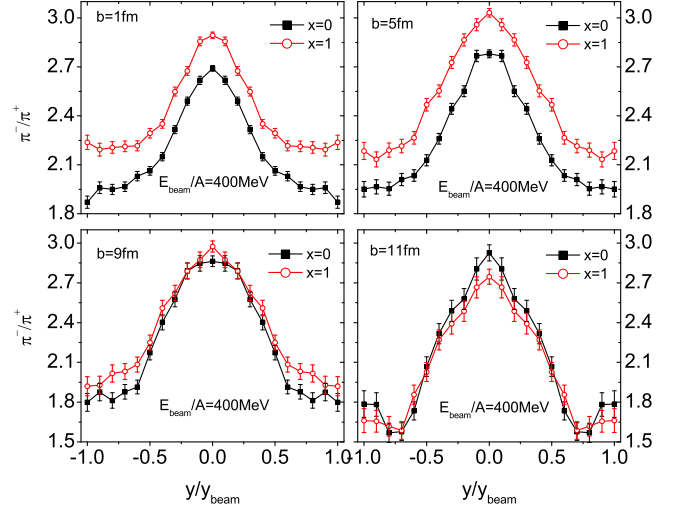


FIG. 3: The rapidity distribution of π^-/π^+ ratio from central to peripheral Pb+Pb collisions at the beam energy of 400 MeV/nucleon.

dynamic pion ratio, i.e., $(\pi^-/\pi^+)_{\text{like}}$, can be defined as

$$(\pi^-/\pi^+)_{\text{like}} \equiv \frac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0}{\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+}. \quad (2)$$

Due to all the Δ resonances will eventually decay at the final reaction stage, it is thus the $(\pi^-/\pi^+)_{\text{like}}$ ratio will naturally become the π^-/π^+ ratio. Shown in Figs. 2 and 3 are the time evolution of $(\pi^-/\pi^+)_{\text{like}}$ ratio and rapidity distribution of π^-/π^+ ratio from central to peripheral Pb+Pb collisions at the beam energy of 400 MeV/nucleon. Similar to previous results [22, 24], the dynamic $(\pi^-/\pi^+)_{\text{like}}$ ratio and final π^-/π^+ ratio

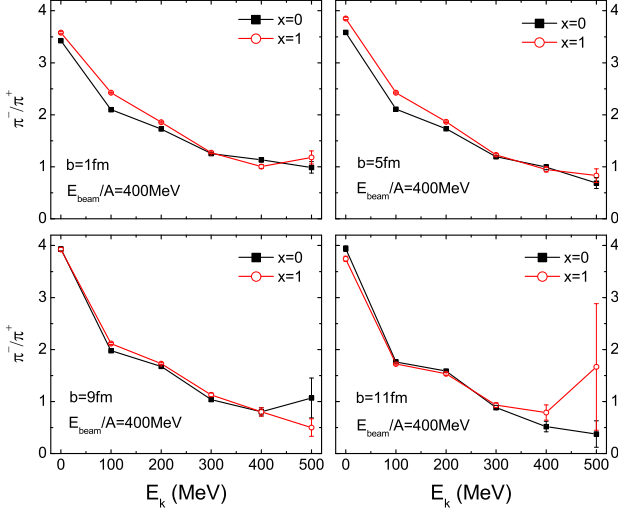


FIG. 4: The kinematic energy distribution of π^-/π^+ ratio from central to peripheral Pb+Pb collisions at the beam energy of 400 MeV/nucleon.

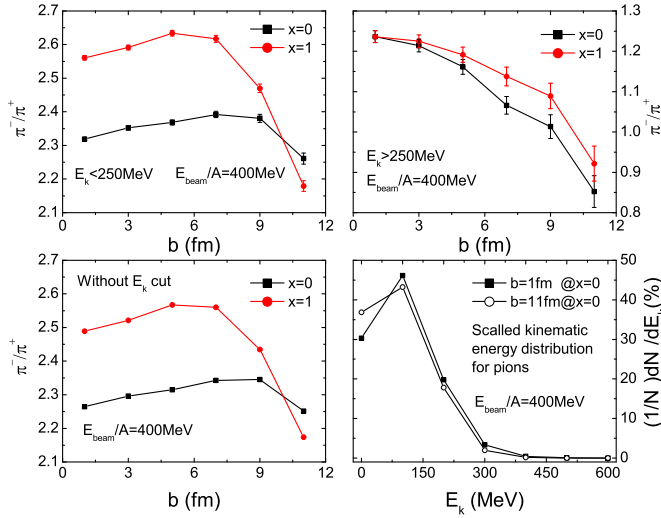


FIG. 5: The impact parameter dependence of π^-/π^+ ratio formed by pions with kinematic energy lower than 250 MeV, larger than 250 MeV, and without any kinematic energy cut from Pb+Pb collisions at the beam energy of 400 MeV/nucleon; and the kinematic energy distribution of pion number percentage from Pb+Pb collisions with symmetry energy parameter $x=0$ at two impact parameters of 1 fm and 11 fm and the beam energy of 400 MeV/nucleon.

are more sensitive to the symmetry energy at the central heavy-ion collision compared to the case of peripheral heavy-ion collision, and larger with a softer symmetry energy setting $x=1$ compared to the case of the stiffer symmetry energy setting $x=0$. However, it can be seen that no matter the dynamic $(\pi^-/\pi^+)_{\text{like}}$ ratio or the rapidity distribution of final π^-/π^+ ratio on the symmetry energy show a tran-

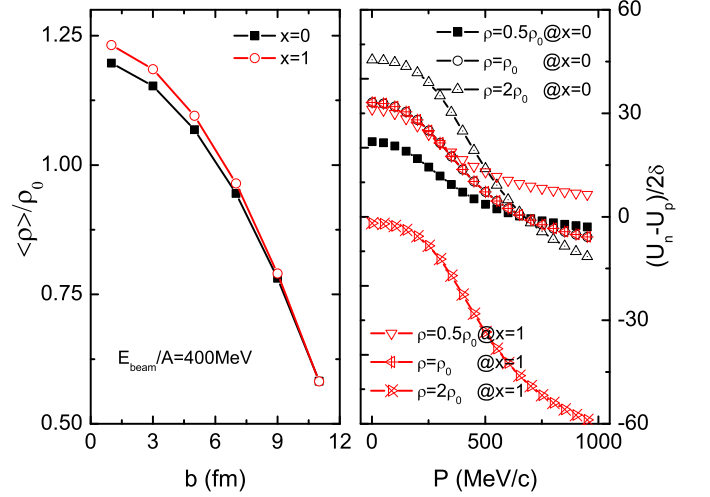


FIG. 6: The average density of participating region over the whole reaction time from central to peripheral Pb+Pb collisions at the beam energy of 400 MeV/nucleon; and the momentum dependence of symmetry potential at different nuclear matter density.

sition from central to peripheral collisions, i.e., the stiffer symmetry energy leads to a larger π^-/π^+ ratio in peripheral collisions while the softer symmetry energy obviously gets this ratio to be larger in central collisions. On the other hand, it is well known that the increasing of impact parameter will directly change the participant numbers, and thus the pion multiplicities and kinematic energy distribution. Therefore, a natural question is which energy range of pion does invert the sensitivity of π^-/π^+ ratio to symmetry energy from central to peripheral collisions. To this end, we show in Fig. 4 the kinematic energy distribution of π^-/π^+ ratio with different impact parameter. In general, it is similar to above observation the sensitivity of π^-/π^+ ratio to symmetry energy is decreasing at lower kinematic energy as increasing the impact parameter especially from midcentral to peripheral collisions, and even shows a opposite sensitivity in very peripheral collisions. However, for the pion ratio at larger kinematic energy, its value with the softer symmetry energy is also larger even in very peripheral collisions albeit with a larger error bar. This naturally gets us to look at the impact parameter dependence of π^-/π^+ ratio formed by less energetic pions and more energetic pions, separately. For this purpose, we take empirically a kinematic energy cut of 250 MeV and classify pions into less energetic and more energetic groups. Shown in upper panel of Fig. 5 are the impact parameter dependence of π^-/π^+ ratio formed by less energetic pions and more energetic pions, respectively. It is seen that the transition of sensitivity of π^-/π^+ ratio to symmetry energy is mainly from less energetic pions, i.e., the softer symmetry energy gets the less energetic pions to form a smaller π^-/π^+ ratio in peripheral collisions while these pions generate a larger

π^-/π^+ ratio in central collisions. Certainly, the softer symmetry energy also leads more energetic pions to form a larger π^-/π^+ ratio in peripheral collisions. Nevertheless, due to most of pions are less energetic at this beam energy as shown in the right plot at lower panel of Fig. 5, thus the behaviour of π^-/π^+ ratio formed by all pions without any kinematic energy cut is almost similar to those formed by less energetic pions as shown in the left plot at lower panel of Fig. 5.

Now let us to show the reason of pion ratio transition in probing the symmetry energy from central to peripheral collisions. To this end, we show the average density of participating region over the whole reaction time in the left panel of Fig. 6 from central to peripheral collisions. It can be found that almost all the pions are produced at supersaturation density at central heavy-ion collisions but subsaturation density at peripheral collisions. On the other hand, from the symmetry potential in right panel of Fig. 6 and symmetry energy in Fig. 1, it can be found that the stiffer symmetry energy with parameter $x=0$ generates a larger symmetry energy and a larger symmetry potential when the density of participating region is higher than the normal density, thereby generating a stronger repulsive effects for neutrons but attractive effects for protons and thus leading to a smaller π^-/π^+ ratio in central collisions. On the contrary, when the density of participating region is lower than the normal density, the stiffer symmetry energy with parameter $x=0$ corresponds to a smaller symmetry energy and a smaller symmetry potential compared to the case of the softer symmetry energy with parameter $x=1$, and then

naturally generating a larger π^-/π^+ ratio in peripheral collisions. This implies the π^-/π^+ ratio as a probe of high-density symmetry energy effective only in central at most to midcentral collisions, thereby avoiding the possible information of low-density symmetry energy carried in π^-/π^+ ratio from peripheral collisions.

In summary, we have carried out an investigation about the impact parameter dependence of π^-/π^+ ratio in probing the nuclear symmetry energy using heavy-ion collision. Within an isospin-dependent transport model, the Pb+Pb collisions are performed with different impact parameter at a beam energy of 400 MeV/nucleon. It is shown that the sensitivity of π^-/π^+ ratio on symmetry energy has a transition from central to peripheral collisions due to the less energetic pions measure the high-density symmetry energy in central collisions but the low-density symmetry energy in peripheral collisions. Therefore, we suggest the π^-/π^+ signature as a high-density symmetry probe effective only in central at most to midcentral collisions. Certainly, other effects such as pion production threshold and energy conservation and pion potential, which are not considered in the present study, can also influence significantly the sensitivity of π^-/π^+ ratio to symmetry energy as shown in others [31–33].

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